Period 1, April 25, 2025

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

$\frac{dH}{dT} = 0$

$\frac{1}{2}(H-1)\cos\frac{T}{2} = 0$

$H > 1$ never be zero

$\cos\frac{T}{2} = 0$

$\cos\frac{-\pi}{2} = \cos\frac{\pi}{2} = \cos\frac{3\pi}{2} = \cos\frac{5\pi}{2} = 0$

$5 > H > 1$

always +

$\frac{1}{2}(H-1)\cos\frac{T}{2} = \frac{dH}{dT} =$ Slope of $H(T)$

$T = \pi = 3.14159$

$\frac{dH}{dT} =$ + + + + + + π - - - - - -

0 1 2 3 4 5

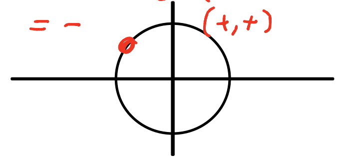
$\frac{\pi}{2} = 90^\circ$

$\frac{\pi}{2} = 1.57$

$\frac{\pi}{2} = 1.57$ (cos, sin)

(+, +)

cos = −



$\cos 0 = 1$

$\cos\frac{4}{2} = \cos 2 = -$

Use separation of variables to find $y = H(t)$, the particular solution to the differential equation

$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

$T = 0 \quad H = 4$

$$\cancel{dT}\,\frac{dH}{\cancel{dT}} = \frac{\frac{1}{2}\cancel{(H-1)}\cos\frac{T}{2}}{\cancel{H-1}}\cdot dT$$

$H-1$

$$\int \frac{1}{H-1}\,dH = \int \frac{1}{2}\cos\frac{T}{2}\,dT$$

$u = \frac{T}{2} = \frac{1}{2}T$

$u = H-1$

$du = dH$

$du = \frac{1}{2}dT$

$2du = dT$

$$\int \frac{1}{u}\cdot du = \int \cancel{\tfrac{1}{2}}\cos u \cdot \cancel{2}\,du$$

$$\ln|u| = \sin u + c$$

$$\ln|H-1| = \sin\frac{T}{2} + c$$

$H > 1$

Always +

$x^2 \cdot x^3 = x^{2+3} = x^5$

$\ln(H-1) = \sin\frac{T}{2} + c$

$\ln(4-1) = \sin\frac{0}{2} + c$

$\sin 0$

$\ln 3 = 0 + c$

$\ln 3 = c$

$\ln(H-1) = \sin\frac{T}{2} + \ln 3$

$\ln_e a = b \Rightarrow e^b = a$

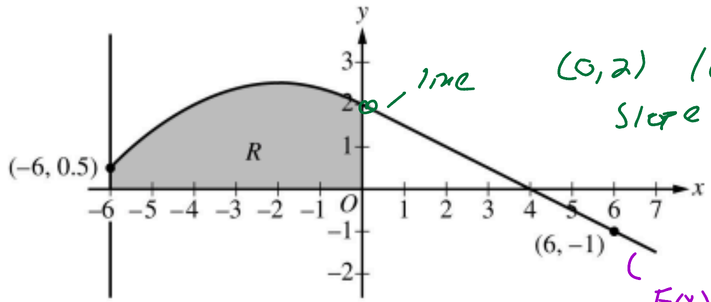
$H - 1 = e^{\sin\frac{T}{2} + \ln 3}$ $+1$ $+1$

$H = e^{\sin\frac{T}{2} + \ln 3} + 1$

$H = e^{\sin\frac{T}{2}} \cdot e^{\ln 3} + 1$

$H = e^{\sin\frac{T}{2}} \cdot 3 + 1$

$H = 3e^{\sin\frac{T}{2}} + 1$



Graph of f

(0,2) (6,−1)

Slope

$\frac{-1-2}{6-0} = \frac{-3}{6} = -\frac{1}{2}$

$F(x) = -\frac{1}{2}x + 2$ when $x > 0$

The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

$h(x) = F(x) - \underset{\text{constant}}{\overset{0.5}{F(-6)}} \Rightarrow h'(x) = F'(x) - 0$

The function h is defined by $h(x) = \int_{-6}^{x} f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

$h(6) = \int_{-6}^{6} F'(t)\,dT = F(T)\Big[_{-6}^{6} = F(6) - F(-6) = -1 - 0.5 = -1.5$

$h'(x) = F'(x)$ $F'(6) =$ Slope of $F(x)$ at $x = 6$

$h'(6) = -\frac{1}{2}$

$h''(x) = F''(x)$ $h''(6) = F''(6) =$ concavity of $F(x)$ at $x = 6$

Nope

$F(x) = -\frac{1}{2}x + 2$

$F'(x) = -\frac{1}{2}$

$F''(x) = 0$

$F''(6) = 0$

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

$\frac{dy}{dx} = 0$ and $y = 1$

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$m = 0$

$0 = \frac{-2x}{3 + 4 \cdot 1} \Rightarrow 0 = \frac{-2x}{7}$ $x = 0$

$0^2 + 3(1) + 2(1)^2 = 48$

$5 \neq 48$

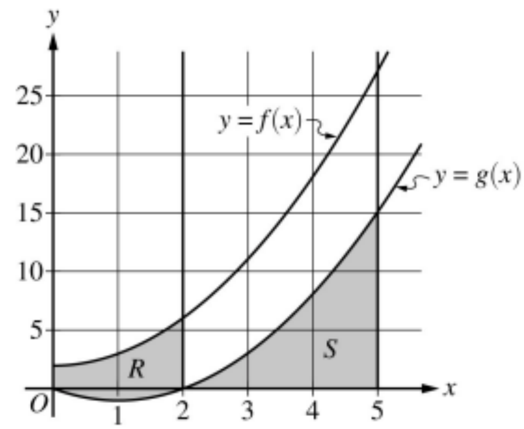
No point on curve That is Tangent To $y = 1$

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

$(\sqrt{48}, 0)$ $\frac{dy}{dx} = \phi = \frac{\text{constant}}{0}$

$(\sqrt{48})^2 + 3(0) + 2(0)^2 = 48$

$\frac{dy}{dx} = \frac{-2 \cdot \sqrt{48}}{3 + 4 \cdot 0} = \frac{-2\sqrt{48}}{3}$ = Nope



The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

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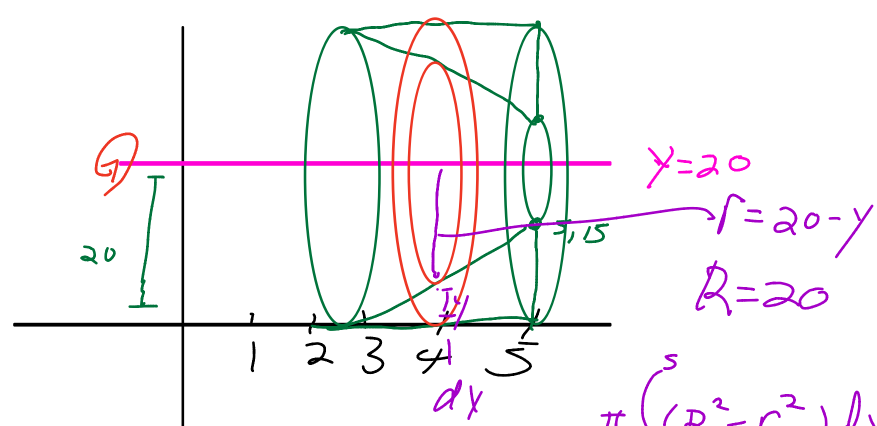

$$\pi\int_2^5 (R^2 - r^2)\,dx$$

$20-(x^2-2x)$

$$\pi\int_2^5 \left(20^2-(20-x^2+2x)^2\right)dx$$

$$\int \frac{x^2}{4} dx = \int \frac{1}{4} x^2 dx = \frac{1}{4} \int x^2 dx = \frac{1}{4} \cdot \frac{1}{3} \cdot x^{2+1=3} + c$$

$$\frac{1}{12} x^3 + c$$